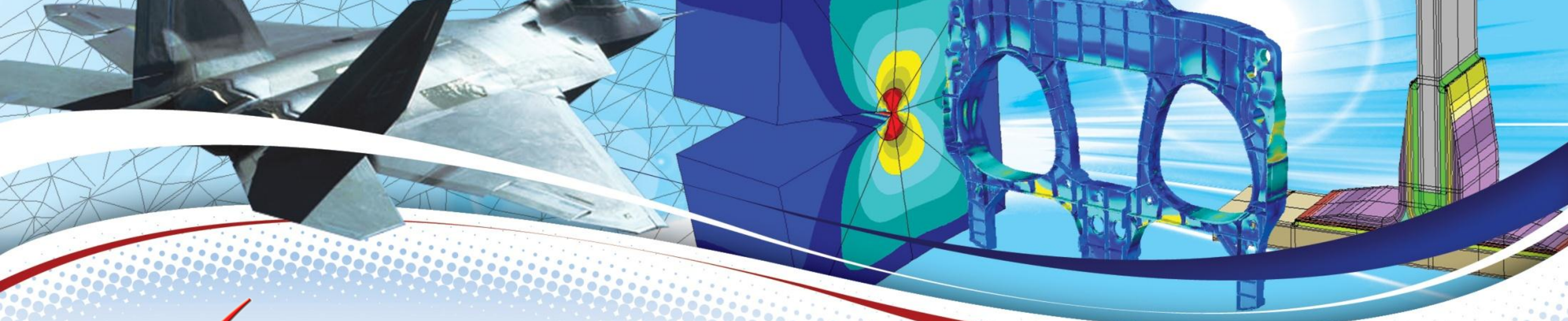
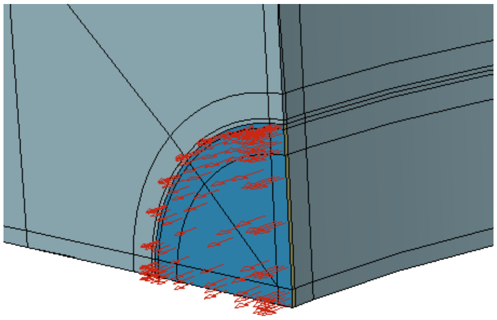


Fracture Analysis with Residual Stresses

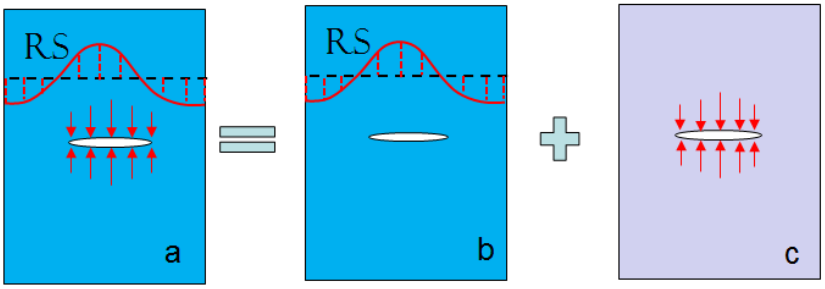
Reliable prediction of crack growth and residual strength in metallic structures requires accurate computation of stress intensity factors. StressCheck® 10.1 enhances the best-in-class fracture analysis capabilities of StressCheck 9.2 by providing the computation of both the separated J-integral components J_I, J_{II}, J_{III}, and the corresponding stress intensity factors K_I, K_{II}, K_{III} for cracks in the presence of residual stresses.

Contour Integral Method with a Loaded Crack (CIM-LC)

As it is, in general, difficult to measure all six components of residual stress throughout a body, the robust contour integral method has been expanded to compute K_I with only the minimal required residual stress information*. If the residual stress component normal to the crack face is known, it can be applied as a crack face pressure; any residual stress distribution can be represented by a formula as a function of local in-plane spatial variables and applied to the crack face.



Formula-defined residual stress load applied to corner crack face of a finite element mesh



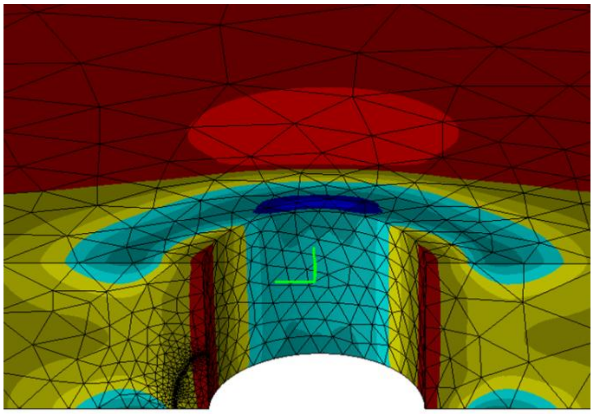
$$K_{1a} = K_{1b} + K_{1c} = 0 \rightarrow \mathbf{K_{1b} = -K_{1c}}$$

J-Integral for Residual Stresses

If the full residual stress distribution is known, the J-integral can be used to calculate **J_I, J_{II}, and J_{III}****. New in StressCheck 10.1 is the capability to convert J into K, providing **K_I, K_{II}, and K_{III} in a residual stress field**.

$$J = \int_{\Gamma} \left(W\delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right) n_i \, ds + \int_{A} \sigma_{ij} \frac{\partial \varepsilon_{ij}^0}{\partial x_1} dA = J_{\Gamma} + J_A$$

$$K_I = \sqrt{\tilde{E} J_I} \qquad K_{II} = \sqrt{\tilde{E} J_{II}} \qquad K_{III} = \sqrt{2G J_{III}}$$



Corner crack defined by a spline in a residual stress field

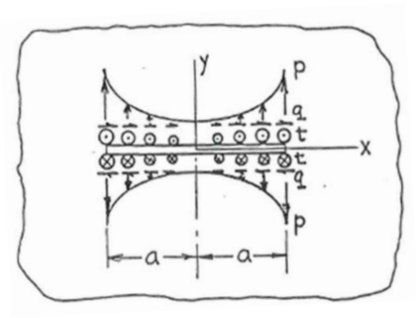
*Pereira, J.P. and Duarte, C.A. (2006). *The Contour Integral Method for Loaded Cracks*, Communication in Numerical Methods in Engineering, Volume 22, pp 421-432. DOI: 10.1002/cnm.824

**Lei, Y., O'Dowd, N.P. and Webster, G.A. (2000). *Fracture Mechanics Analysis of a Crack in a Residual Stress Field*, International Journal of Fracture, Volume 106, pp 195-216.

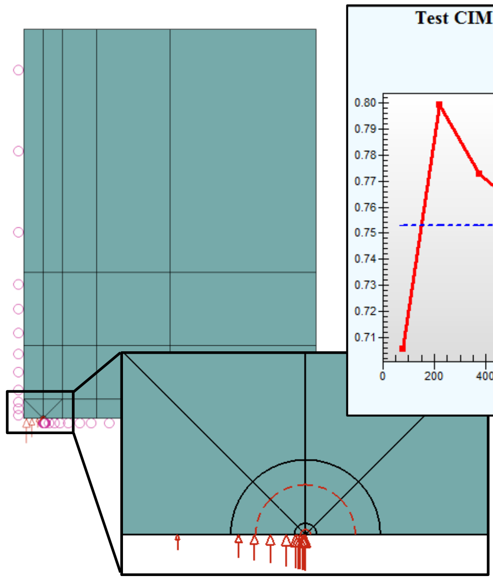
***Tada, H. *Stress Analysis of Cracks Handbook*, Case 11.1.

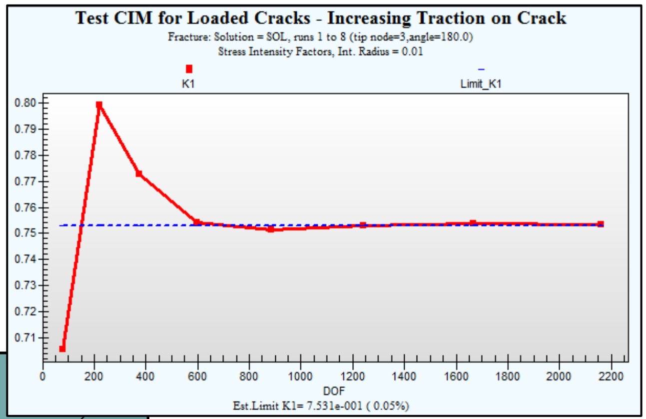
Comparison with Tada Handbook

Published exact solutions for stress intensity factors with loaded crack faces are available in the Tada Handbook***, providing a benchmark for tools that compute stress intensity factors. The StressCheck solution matched the published solution for all cases considered. For example, consider a loaded crack with increasing pressure toward the crack tip.



Infinite plate solution: K_I = **0.752**

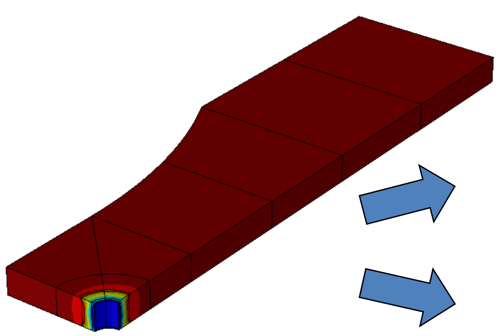




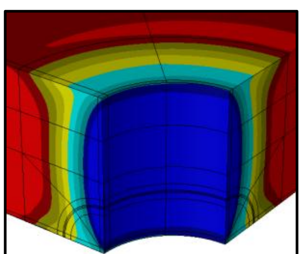
StressCheck finite size plate solution: K_I = **0.753**

Comparison of J-Integral and Contour Integral Method

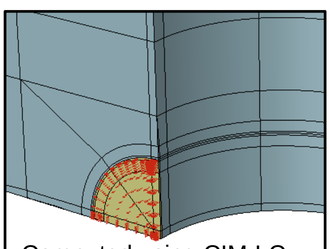
The StressCheck Incremental Plasticity module was used to simulate the coldworking process of a fatigue test specimen, producing the complete residual stress tensor throughout the specimen. The residual stresses were then transferred to a body with a 0.025” corner crack, and used as input for linear analyses leading to stress intensity factor computation by the J-integral and by the contour integral method with a loaded crack face. Both methods provided results which are very close to each other.



Simulated cold working (no crack)



Computed using J-integral



Computed using CIM-LC