The Girkmann problem

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Introduction

There is a growing interest in procedures for ensuring the reliability of predictions based on computed information. For predictions to be reliable, the mathematical model must account for those aspects of the physical reality being modeled that have significant influence on the data of interest and the numerical solution of the mathematical model must be shown to be sufficiently accurate for the purposes of analysis. General guidelines pertaining to the use of mathematical models in solid mechanics were issued by the American Society of Mechanical Engineers in 2006 [1] and adopted by the American National Standards Institute. We are interested in the question of how verification procedures are applied in specific situations in professional practice.

The model problem described herein was first discussed by Girkmann [2], subsequently by Timoshenko and Woinowski-Krieger [3]. Solutions by classical methods are presented in both references. Numerical solution of this problem is presented in [4]. We propose this problem as a benchmark for evaluating verification procedures that an experienced user of a finite element analysis software product would apply in professional practice to verify that the computed information is sufficiently accurate. We invite readers to analyze this problem and send their solutions and comments to the corresponding author to be received on or before 1 June 2008. We will summarize the results and the comments received, without attribution, from the point of view of how the requirements for verification were met.

Problem description

Geometry: The notation is shown in Fig. 1. The *z* axis is the axis of rotational symmetry. A spherical shell of thickness h=0.06 m, crown radius $R_c = 15.00$ m is connected to a stiffening ring at the meridional angle $\alpha = 2\pi/9$ (40^o). The dimensions of the ring are: a = 0.60 m, b = 0.50 m. The radius of the mid-surface of the spherical shell is $R_m = R_c/\sin\alpha$.

Material: Reinforced concrete, assumed to be homogeneous, isotropic and linearly elastic with Young's modulus E=20.59 GPa and Poisson's ratio $\nu = 0$.

Loading:

a) Gravity loading. The equivalent (homogenized) unit weight of the material comprised of the shell and the cladding is 32.69 kN/m^3. In the classical solutions the distributed

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load $T_z = -1.9614$ kN/m^2 was assumed to act on the mid-surface of the shell in the negative z-direction and the ring was assumed to be weightless. In the numerical solution volume forces may be applied to the shell and the stiffening ring.

b) Uniform normal pressure *p* acting at the base AB of the stiffening ring. The resultant of *p* equals the weight of the structure.

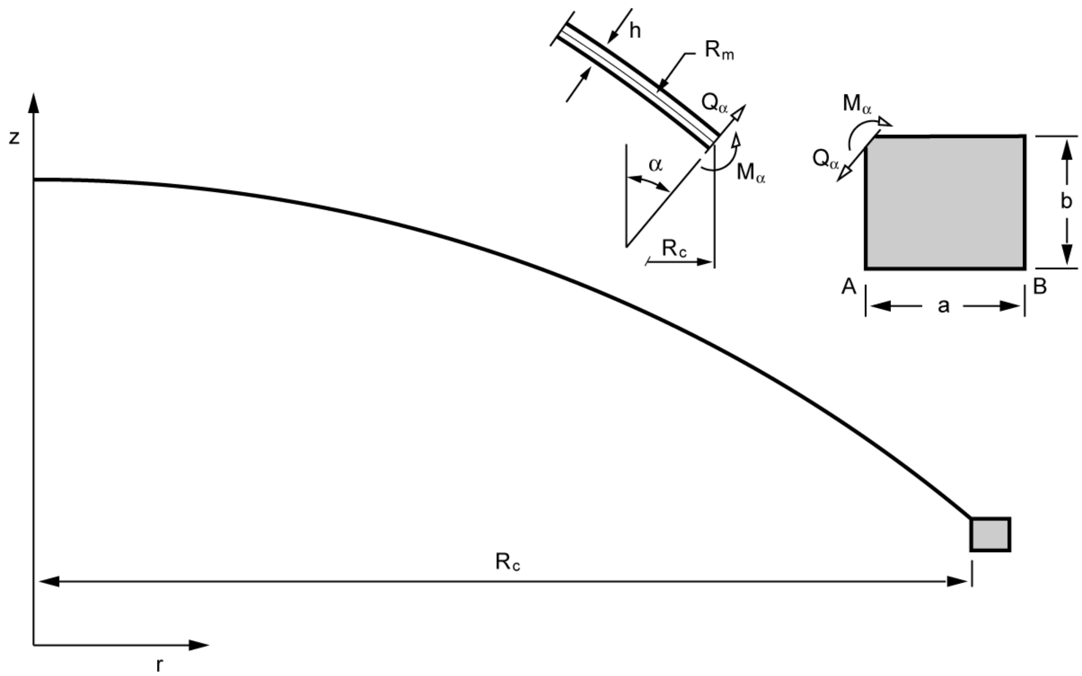

Figure 1: Notation.

Problem statement

The domain, loading and support conditions are axisymmetric. We realize that in engineering applications axisymmetric models are rarely used. It will be acceptable to solve the problem on a sector of the stiffened shell, using symmetry conditions.

a) Find the stress resultants Q_α (shearing force, kN/m units) and M_α (bending moment, Nm/m units) acting at the junction between the spherical shell and the stiffening ring (see Fig. 1).
b) Determine the location (meridional angle) and the magnitude of the maximum bending moment in the shell.
c) Verify that the results are accurate to within 5 percent.
d) State what software, what mesh and what type of elements were used. Describe how the accuracy of the data was verified.

References

[1] Guide for Verification and Validation in Computational Solid Mechanics. ASME V&V 10-2006. The American Society of Mechanical Engineers, New York, 2006.

[2] K. Girkmann. Flächentragwerke. 4th Ed. Springer Verlag, Wien. 1956.

[3] S. Timoshenko and S. Woinowski-Krieger. Theory of Plates and Shells. 2nd Ed. McGraw-Hill Book Company, Inc. New York 1959 pp. 555-558.

[4] B. Szabó and I. Babuška. Finite Element Analysis. John Wiley & Sons, Inc. New York 1991 pp. 327-332[4].

[4] Correction: On page 331, Table 17.5 in reference [4] the radial force component F_r for the Girkmann/Timoshenko model should be -104.6 lbf/in instead of -8.95 lbf/in.