The problem of verification with reference to the Girkmann Problem

J. Pitkaranta, I. Babuska and B. Szabo

In the January, 2008 issue of IACM Expression we presented a problem, called the Girkmann problem [1], and invited readers to solve that problem, report the data of interest and *verify that the reported data are within 5 percent of their exact counterparts.* The data of interest were: (a) the bending moment at the junction of the shell and the ring (b) the shear force at the junction of the shell and the ring, (c) the maximum bending moment in the shell and (d) its location characterized by the meridional angle.

The primary purpose of this exercise was to assess how analysts, working with commercially available finite element software tools, would meet the requirement of verification, given this problem.

By the term verification we understand a process by which it is verified that the approximate solution differs from the exact solution of the mathematical problem by not more than an acceptable tolerance. In our case the mathematical problem is to determine the data of interest based on the three-dimensional linear elasticity formulation, assuming that the material properties and the loads are as specified in the problem statement. It is possible to prove that the exact solution exists and is unique. This implies that the data of interest are finite.

Since the exact solution of this problem is not known, the error in the numerical solution has to be estimated. It is not easy to obtain guaranteed upper and lower bounds of the error in the data of interest. In practice the best way is to compute a sequence of finite element solutions corresponding to a converging sequence of discretizations. Since the exact solution is independent of the discretization, a necessary condition for the errors in the data of interest to be small is that the data of interest are substantially independent of the discretization. To make a stronger statement, namely that the errors in the data of interest are within a given tolerance, involves extrapolation and judgment that the extrapolated values are sufficiently close to their exact counterparts to justify making that statement. Although this approach will not give guaranteed error bounds, it is very reliable. We emphasize that this method of error estimation is based on the assumption that convergence of the sequence of discretization has been proven. If the sequence of the discretized solutions does not converge then the numerical treatment is not correct even if in some cases it produces credible results.

In a forthcoming paper we will present a detailed description of the verification procedure used to arrive at the following results which, in our professional opinion, are within 2 percent of their exact values:

Bending moment at the shell-ring interface: -36.81 Nm/m
Shear force at the shell-ring interface: 0.9436 kN/m
Maximum bending moment: 255.1 Nm/m
Meridional angle of the location of the maximum bending moment: 38.15 degrees

These results are based on the assumption that the shell is loaded, as described in the problem statement, but the footring is weightless. In order to underline our confidence in the statement that these data are within 2 percent of their exact counterparts, we hereby offer 1000 USD to the first person who presents evidence that the error in any one of the given stress resultants is greater than 2 percent.

We received 15 solutions. Respondents used various models: axisymmetric solids, axisymmetric shell-solid combinations, three-dimensional solids and three-dimensional shell-solid combinations. The data of interest were computed by direct integration, from nodal forces and by extraction procedures. Not all respondents included estimates of the location and magnitude of the maximum bending moment and not all respondents presented details on how verification was performed. Of the 15 solutions received, 4 utilized the p-version of the finite element method for verification, 11 utilized the h-version.

The results based on the p-version were well within the 5 percent tolerance specified in the problem statement and respondents provided demonstration of convergence of the data of interest. On the other hand, the results based on the h-version had a very large dispersion. The reported data were within the allowed tolerance of 5 percent in only two of the eleven cases. These two solutions were generated by means of the same commercial finite element analysis software product and the same analyst who used (a) an axisymmetric shell-solid model and (b) a three-dimensional solid model. This analyst did not present evidence of h-convergence, however stated that the prior information published in [2] was used for the stopping criterion.

One respondent attempted to demonstrate h-convergence for a three-dimensional shell-solid model on one quarter of the spherical shell using six successive mesh refinements. In the sixth refinement 120 million degrees of freedom were used. The sequence of moments corresponding to the six refinements still had not converged but appeared to tend to approximately -225 Nm/m and the shear force appeared to have converged to approximately 11.3 kN/m.

Another respondent wrote: *"Regarding verification tasks for structural analysis software that has adequate quality for use in our safety critical profession of structural engineering, the solution of problems such as the Girkmann problem represents a minuscule fraction of what is necessary to assure quality."* We agree with this statement. That is why we find it very surprising that the answers received had such a large dispersion. For example, the reported values of the moment at the shell-ring interface ranged between -205 and 17977 Nm/m. Solution of the Girkmann problem should be a very short exercise to persons having expertise in FEA, yet many of the answers were wildly off.

The results indicate that the requirements of verification pose challenges that users and vendors of commercial finite element analysis software products should address.

References

[1] K. Girkmann. Flächentragwerke. 4th Ed. Springer Verlag, Wien. 1956.

[2] B. Szabó and I. Babuška. Finite Element Analysis. John Wiley & Sons, Inc. New York 1991 pp. 327-332.